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Mass transfer into turbulent liquid jets

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Abstract—Mass transfer into turbulent liquid jets has been discussed based on the eddy diffusivity model. Consistency of the available experimental data on the mass transfer coefficients for such systems has been examined. For the experimental data which proved to be consistent with the theory the eddy diffusivity parameter has been determined and related to the basic hydrodynamic parameters of the system.

INTRODUCTION

MASS TRANSFER into turbulent liquid jets has almost exclusively been studied by Davies and co-workers [1–3]. More recent data have been published by Ide *et al.* [4]. All these data refer to liquid jets with the jet Reynolds numbers exceeding about 4000. In contrast to the laminar free jets the surface of turbulent jets is not smooth and is covered with protrusions [1, 3, 5]. At jet velocities corresponding to $Re_0 \geqslant 25\,000$ surface atomization of the jet takes place [3]. This effect is diminished if some additives into the water are used [3]. According to Davies and Young-Hoon [3] such an additive ('Polyox' solution) reduces the stratching rates of the protrusions and thus increases the mean radii of the protrusions. Reduced pressure of a gaseous environment does not influence the surface protrusions for jets with lengths up to 70 mm. These observations entitled Davies and Young-Hoon [3] to draw a conclusion that the turbulence structure in the jet and the resulting surface protrusions are actually a visualization of eddies which reach the jet surface and which are entirely produced in the issuing nozzle.

Under these conditions the mass transfer area changes relatively slightly in comparison to the geometrical area of a cylinder with a diameter equal to that of the nozzle and with a length equal to that of the jet. The measurements of some authors [1, 3] demonstrated that the differences between the measured and assumed areas are within $\pm 10\%$ and are approximately within the experimental error of the mass transfer coefficient determinations.

In the mass transfer studies water–CO_2, water–H_2, kerosene–CO_2, H_2 and He and CO_2–water with additives such as polyethylene oxide and a surface tension agent (sodium dioctylsulphosuccinate) were used [1–5]. Nozzles of different design with diameters ranging from 1 to 2 mm i.d. [1–3] and 12 mm i.d. [4] were used in these studies. The experimental data on mass transfer collected by all these authors were interpreted by means of the eddy diffusivity model developed by Levich and Davies, under the assumption of the fully developed concentration profile, and no variation of the jet turbulence parameters along its length [6]. After substitution of the friction velocity following from the Blasius formula they obtained a dimensionless correlation of the form

$$Sh_m = C_1\, Re_0^{21/16}\, Sc^{1/2}[\mu^2/(d_0 \rho \sigma)]^{1/2} \qquad (1)$$

which predicts that $\bar{k}_L$ is proportional to $Re_0^{1.31}$. The experimental evidence seemed to confirm theoretical predictions since the power exponent at Re_0 was found to range from 1.22 to 1.42. However, the values of the constant C_1 varied from 0.020 to 0.033 for the systems water–CO_2 and water–H_2 [1] and equal to about 0.016 for the systems with kerosene [2]. These discrepancies could not be explained properly.

The experimental data of Davies and co-workers were subjected to further discussions by other authors [7–11]. Theofanous and co-workers [8, 9] pointed out that it is not justified to assume that turbulence will remain constant along the jet length and suggested that it is indispensable to account for their decay due to viscous effects. By using data obtained for decay of turbulence downstream of a grid, they obtained better agreement and consistency of the constants in the Levich model of mass transfer. Also application of the eddy-cell model in a version developed by Brumfield and Theofanous [8] and under the assumption that the large-scale eddies are responsible for mass transfer in such a system, yielded relatively good results giving reasonably consistent constants in this model.

Further important points have been raised by Mills *et al.* [10]. These authors noticed that the experimental data of Davies and co-workers on the mass transfer coefficients were surprisingly low. From direct comparison with the predictions of the well-known penetration theory they concluded that the $\bar{k}_L$ values at $Re_0 \leqslant 10^4$ are considerably lower than those resulting from this theory, which suggested substantial systematic error in the experiments. They also noted that these low values of $\bar{k}_L$ at low turbulent Reynolds numbers gave rise to high apparent slopes of $\bar{k}_L$ vs Re_0 in

NOMENCLATURE

- c concentration of the solute [kmol m^{-3}]
- C_i numerical constant in the ith equation
- d_0 nozzle diameter [m]
- D molecular diffusivity [m^2 s^{-1}]
- D_t eddy diffusivity [m^2 s^{-1}]
- k_L local mass transfer coefficient [m s^{-1}]
- $\bar{k}_L$ average mass transfer coefficient [m s^{-1}]
- L jet length [m]
- r radial coordinate [m]
- R jet radius [m]
- Re_0 jet Reynolds number, Vd_0/ν
- Sc Schmidt number, ν/D
- Sh_m mean Sherwood number, $\bar{k}_L d_0/D$
- V bulk velocity [m s^{-1}]
- x axial coordinate [m]
- X dimensionless axial distance, C_4x/V
- Y dimensionless radial coordinate, $1-r/R$
- Z dimensionless variable, $\sqrt{\beta}\,Y$.

Greek symbols
- β dimensionless eddy diffusivity group, C_4R^2/D
- θ dimensionless concentration, $(c_{sat}-c)/(c_{sat}-c_{in})$
- ν kinematic viscosity [m^2 s^{-1}]
- ρ liquid density [kg m^{-3}]
- σ surface tension [N m^{-1}]
- ψ dimensionless group, $k_L\sqrt{(\pi x/(VD))}$
- ψ_m dimensionless group, $\bar{k}_L\sqrt{(\pi x/(VD))}$.

Subscripts
- in bulk value at inlet
- sat saturation value.

the turbulent regime. Therefore, they attempted to get a better insight into the problem by considering the solution of the entrance region problem for a turbulent liquid jet. The details of this attempt and some of our own modifications will be presented in the next paragraph. Now it can only be added that Mills *et al.* [10] tried to numerically solve the appropriate mass balance equation and presented a unique result of these computations based on some selected data points of Davies and Ting [1]. Mills *et al.* failed to reproduce the experimental data of Davies and co-workers because of basic difficulties in avoiding gas entrainment into the outlet stream.

Obot [11] has presented a new correlation of the experimental data of Davies and co-workers based on somewhat different dimensionless numbers. He showed that the jet length is an important parameter, in particular for the lower range of Re_0. A similar method of the experimental data treatment seemed to be equally applicable for turbulent falling liquid films.

Conclusions drawn by Mills *et al.* [10] and by Obot [11] prompted further considerations which would shed more light on the hitherto collected experimental data on mass transfer into turbulent liquid jets.

THEORETICAL CONSIDERATIONS

Mills *et al.* [10] formulated the species conservation equation for a turbulent jet

$$V\frac{\partial c}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(D+D_t)\frac{\partial c}{\partial r}\right] \quad (2)$$

which must be solved subject to the following conditions:

$$x = 0: \quad c = c_{in},$$
$$r = 0: \quad \partial c/\partial r = 0,$$
$$r = R; \quad c = c_{sat}. \quad (3)$$

In equation (2) plug flow can be assumed in a jet of constant diameter and the eddy diffusivity model of mass transfer used to characterize turbulent transport. The first assumption is justified from the experimental data of Davies and Makepeace [12] from which it follows that the surface velocity of a turbulent jet of pure water reaches about 90% of the mean jet velocity in about 3 mm, or in 1.5 jet diameters. On the other hand, the effect of the presence of surface active agents becomes negligible further than 6 mm from the nozzle. The eddy diffusivity can be approximated by an equation which is applicable for turbulent falling films

$$D_t = C_4y^2. \quad (4)$$

This approximation is probably valid only very close to the interface. Since the numerical calculations given by Mills *et al.* [10] revealed that the concentration boundary layer hardly penetrates into the region of the jet where turbulent transport is significant, the form of equation (4) should be appropriate for further derivations. It is convenient to introduce a dimensionless form of equation (2) by using the following representation of dimensionless variables

$$\theta = \frac{c_{sat}-c}{c_{sat}-c_{in}}; \quad Y = 1-r/R; \quad X = C_4x/V;$$
$$\beta = C_4R^2/D.$$

Now equation (2) becomes

$$\beta\frac{\partial\theta}{\partial X} = \frac{1}{1-Y}\frac{\partial}{\partial Y}\left[(1-Y)(1+\beta Y^2)\frac{\partial\theta}{\partial Y}\right] \quad (5)$$

with the boundary conditions as follows:

$$\theta(0,Y) = 1; \quad \theta(X,0) = 0; \quad Y = 1; \quad \partial\theta/\partial Y = 0. \quad (6)$$

Equation (5) can be simplified if the depth of the solute penetration is small enough, i.e. $Y \ll 1$. With this simplification equation (5) becomes

$$\beta \frac{\partial \theta}{\partial X} = \frac{\partial}{\partial Y}\left[(1+\beta Y^2)\frac{\partial \theta}{\partial Y}\right]. \quad (7)$$

Following a method of solution of a similar equation discussed in an earlier paper [14], a substitution of a new variable $Z = \sqrt{\beta} Y$ can be used. This transforms equation (7) into

$$(1+Z^2)\frac{\partial^2 \theta}{\partial Z^2} + 2Z\frac{\partial \theta}{\partial Z} = \frac{\partial \theta}{\partial X}. \quad (8)$$

Exactly the same equation has been solved by Gottifredi and Quiroga [15] for the whole range of X for the case of mass transfer into turbulent film flow. Other solutions of such an equation are discussed in ref. [13]. From the solution given by Gottifredi and Quiroga the local values of the mass transfer coefficients can be obtained. It is convenient to introduce a dimensionless term, ψ, and an expression for ψ resulting from the Gottifredi and Quiroga solution can be obtained as follows:

$$\psi = (\pi X)^{1/2}\left\{\frac{\exp(-4X/\pi^2)}{(\pi X)^{1/2}} + \frac{2}{\pi}\,\mathrm{erf}\,[4/(\pi^2 X)^{1/2}]\right\}. \quad (9)$$

For small values of X equation (9) simplifies into

$$\psi \cong 1 + \frac{4X}{\pi^2} - \frac{8X^2}{3\pi^4} + \frac{32X^3}{15\pi^6} + \frac{32X^4}{7\pi^8} + \ldots \quad (10)$$

For $X \leqslant 1$ the error involved with application of the approximate solution given by equation (10) is less than 0.05%, whereas at $X = 2$ this error is about 0.6%.

From the practical point of view the average values of the mass transfer coefficients are of interest since they can directly be comparable with the experimental data. Integration over the jet length gives an expression for the mean Sherwood number of ψ_m based on the average mass transfer coefficient. The values of ψ_m have been given in Table 1 of ref. [13]. The classical penetration solution gives $\psi_m = 2$, whereas the long contact time asymptote is practically attained at $X \geqslant 20$ and corresponds to

$$\psi_m = \frac{2}{\sqrt{\pi}} X^{1/2}. \quad (11)$$

For the range of $X \leqslant 1$ a simple approximation can be used for ψ_m

$$\psi_m = 2 + 0.262 X^{0.99187} \quad (12)$$

with an error smaller than 0.1% with respect to the accurate values listed in Table 1 of ref. [13]. The theoretical dependence of ψ_m vs X is shown in Fig. 1.

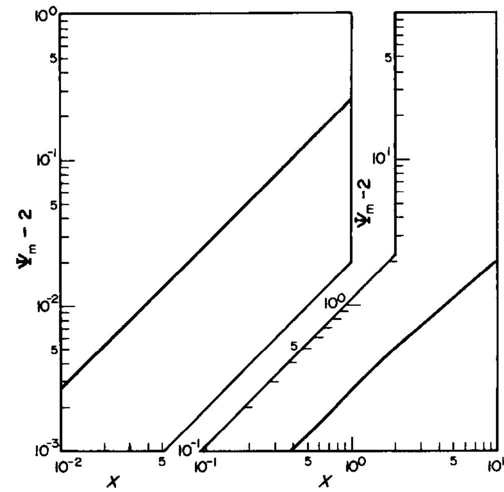

FIG. 1. Dependence of ψ_m on the dimensionless axial distance, X.

DISCUSSION

Mills *et al.* [10] solved numerically equation (2). They reported the results of their computations for only one set of the process parameters taken from the experiments of Davies and Ting [1], i.e. for $d_0 = 1.51$ mm, $V = 8.64\,\mathrm{m\,s^{-1}}$ and $Re_0 = 15000$. In a graphical form they gave the variation of the local as well as the average values of the mass transfer coefficients along the jet length and also the calculated eddy diffusivity and concentration profiles. From these data the values of ψ and ψ_m can easily be determined. Having established the values of ψ_m the dimensionless distance, X, can be determined from equation (12) or Fig. 1, and the eddy diffusivity parameter, C_4, then calculated. The results of such a procedure are given in Table 1. They refer to the jet length of $L = 0.08$ m. As it is seen in this table the eddy diffusivity parameter varies from 43 to $112\,\mathrm{s^{-1}}$, with an average value of about $66 \mp 20\,\mathrm{s^{-1}}$. Using this average value of C_4, from the back-calculations the values of ψ_m and ψ, and then accordingly $\bar{k}_L$ and k_L can be computed and compared with the data given by Mills *et al.* [10]. It can be concluded that the values of $\bar{k}_L$ averaged over the jet length are not particularly sensitive to the accuracy of the determination of the C_4 values, whereas the local values of k_L are much more sensitive to this parameter. The accuracy of predictions of k_L based on the averaged value of C_4 is about 20% at the nozzle and improves considerably at the end of the jet.

Another verification of the simplified theory presented in the previous section can be obtained from a comparison of the eddy diffusivity data distribution resulting from the numerical computations of Mills *et al.* [10] (cf. their Fig. 3) and that calculated from the established average value of the eddy diffusivity parameter. Figure 2 shows that the curve obtained from the present computations has a similar trend as those resulting from numerical calculations of Mills

Table 1. Values of the eddy diffusivity parameter, C_4, estimated from the data of Mills *et al.* [10]

x/L	$\bar{k}_L \times 10^4$† (m s^{-1})	ψ_m	X	C_4 (s^{-1})	$\bar{k}_L \times 10^4$‡ (m s^{-1})	$k_L \times 10^4$‡ (m s^{-1})	$k_L \times 10^4$† (m s^{-1})
0.065	21.0	2.0070	0.026	43	21.0	10.63	—
0.100	17.0	2.0152	0.057	62	17.0	8.64	10.75
0.200	12.3	2.0553	0.208	112	12.1	6.26	7.48
0.400	8.73	2.0698	0.264	71	8.71	4.63	5.16
0.600	7.20	2.0907	0.344	62	7.22	3.94	4.20
0.800	6.32	2.1190	0.451	61	6.35	3.55	3.61
0.885	6.00	2.1159	0.439	54	6.08	3.44	—
1.000	5.73	2.1480	0.562	61	5.76	3.30	3.24

† Taken from Fig. 2 in ref. [10].
‡ Calculated based on the average value of $C_4 = 66\,s^{-1}$.

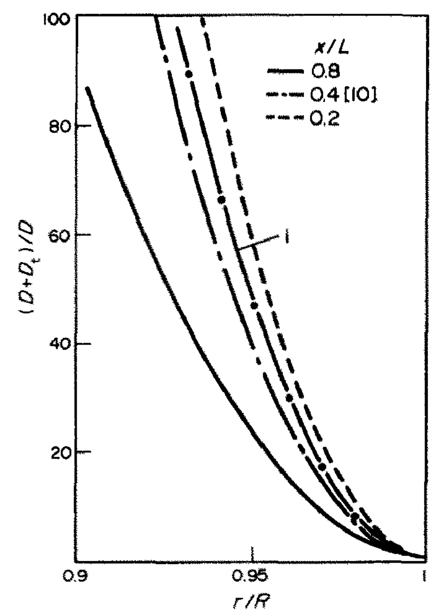

FIG. 2. Calculated eddy diffusivity profiles in a turbulent water jet (after Mills *et al.* [10]): 1, own curve with $C_4 = 66\,s^{-1}$.

et al., although the latter reflect distinct dependence on the axial coordinate, x.

It can be concluded then that the eddy diffusivity model of mass transfer in the version developed earlier for the falling turbulent liquid films proved also to be applicable for turbulent liquid jets. Furthermore, the theoretical considerations presented in this paper provide a better insight in the understanding of the effect of the basic hydrodynamic parameters of the jet on the mass transfer coefficients in a more comprehensive form.

In order to determine the values of the eddy diffusivity parameter, C_4, the experimental data of Davies and Ting [1], Davies and Young-Hoon [3], and that of Ide *et al.* [4] have been considered. Unfortunately, the data of Davies and Hameed [2] could not be used since there is no distinction between the data points for different nozzles, different jet lengths and different gases being absorbed, indicated on their graph. From the experimental data on the average mass transfer coefficients the values of ψ_m could be calculated.

As it follows from equation (12) or Fig. 1, the minimum value of ψ_m should equal 2 according to the present solution of equation (2). Thus the calculated values of $\psi_m < 2$ suggest a substantial systematic error in experiments, as it has been pointed out by Mills *et al.* [10]. This seems actually to be the case for the experimental data points given by Davies and co-workers [1, 3] for which $Re_0 < 11\,000$ and $L \leqslant 0.06$ m. All data points given by Ide *et al.* [4] yield the values of $\psi_m > 2$. For all the data points for which $\psi_m > 2$, the values of X could be obtained either from equation (12) or directly from Fig. 1. Now, using the definition of the term X, the values of the eddy parameter, C_4, can readily be determined, as it has been demonstrated in Table 1 for the data of Mills *et al.* [10]. The results of such calculations are shown in Fig. 3 which gives a dependence of the eddy diffusivity parameter, C_4, on the jet Reynolds number, Re_0. The straight lines drawn in Fig. 3 have a slope close to 3. A dependence on the jet length, L, is also evident. If a dimensionless correlation for the eddy diffusivity, C_4, is attempted, an expression of the form

$$\frac{C_4 d_0^2}{\nu} = C_{13}(L/d_0)^{-2}\,Re_0^3 \qquad (13)$$

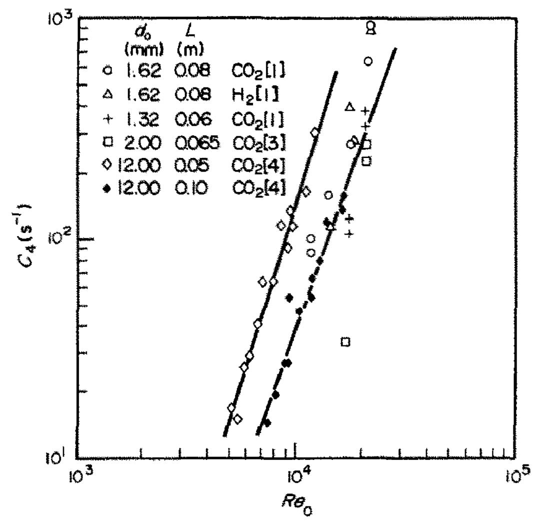

FIG. 3. Correlation of the eddy diffusivity parameter, C_4, with the jet Reynolds number, Re_0.

can be obtained with $C_{13} = 4 \times 10^{-7}$ and with an average error of 26% for the data of Davies and Ting [1] and Ide *et al.* [4]. The overall accuracy of determination of the average values of the mass transfer coefficients based on equation (13) and using Fig. 1 is about 5.3%. The error involved in determination of the values of $\bar{k}_L$ is not particularly sensitive to the accuracy with which values of C_4 are known. For example, a 100% error in C_4 leads to an error in $\bar{k}_L$ not greater than 5% for the range of $X \leqslant 1$, and not greater than about 15% for $X \leqslant 4$.

The main problem in analysing the experimental data published so far is their reliability. It is not yet clear enough why some data due to Davies and coworkers [1, 3] are consistent giving the values of $\psi_m \geqslant 2$, and some data of these authors fall into a region of significant deviation from the simple penetration theory predictions. Obviously, more experimental evidence would be desirable to confirm the application of the theory presented above.

CONCLUSIONS

(1) Mass transfer into turbulent liquid jets has been theoretically considered and a simplified solution of an appropriate differential equation based on eddy diffusivity concept obtained.

(2) Consistency of the available experimental data has been checked with the aid of the developed theory. For the data which proved to be consistent with the theory ($\psi_m > 2$), the eddy diffusivity parameter has been estimated and correlated with the main hydrodynamic parameters of the system.

(3) More reliable experimental data on the mass transfer coefficients in the turbulent jet systems are required to check the validity of the theory presented in the paper, originally developed for turbulent falling films.

REFERENCES

1. J. T. Davies and S. T. Ting, Mass transfer into turbulent jets, *Chem. Engng Sci.* **22,** 1539–1548 (1967).
2. J. T. Davies and A. Hameed, Gas absorption into turbulent jets of kerosene, *Chem. Engng Sci.* **26,** 1295–1296 (1971).
3. J. T. Davies and A. A. Young-Hoon, Restrained turbulent jets of a non-Newtonian solution, *Chem. Engng Sci.* **29,** 1115–1121 (1974).
4. M. Ide, J. Mada and H. Shinohara, Gas absorption into turbulent liquid jet, *Kyushu Daigaku Kogaku Shuho* **52,** 499–504 (1979).
5. J. T. Davies, Interfacial effects in gas transfer to liquids, *Chem. Ind.* 189–193 (1980).
6. J. T. Davies, *Turbulence Phenomena.* Academic Press, New York (1972).
7. F. Coeuret and J. J. Ronco, Absorption physique d'un gaz dans un jet liquide turbulent, *Chem. Engng Sci.* **28,** 315–318 (1973).
8. L. K. Brumfield and T. G. Theofanous, Turbulent mass transfer in jet flow and bubble flow: a reappraisal of Levich's theory, *A.I.Ch.E. Jl* **22,** 607–610 (1976).
9. T. G. Theofanous, R. N. Houze and L. K. Brumfield, Turbulent mass transfer at free gas–liquid interfaces, with applications to open-channel, bubble and jet flows, *Int. J. Heat Mass Transfer* **19,** 613–624 (1976).
10. A. F. Mills, S. Kim, T. Leininger, S. Ofer and A. Pesaran, Heat and mass transfer in turbulent liquid jets, *Int. J. Heat Mass Transfer* **25,** 889–897 (1982).
11. N. T. Obot, Analysis of gas absorption by turbulent liquid jets, *Can. J. Chem. Engng* **61,** 250–253 (1983).
12. J. T. Davies and R. W. Makepeace, Measurement of the surface ages of water jets, *A.I.Ch.E. Jl* **24,** 524–530 (1978).
13. A. K. Biń, Mass transfer into a turbulent liquid film, *Int. J. Heat Mass Transfer* **26,** 981–991 (1983).
14. A. Biń, Mass transfer into a turbulent radial liquid flow, *Inż. Chemiczna Procesowa* **10,** 21–30 (1980).
15. J. C. Gottifredi and O. D. Quiroga, Gas absorption into a turbulent falling film, *Chem. Engng Jl* **16,** 199–204 (1978).

TRANSFERT DE MASSE DANS LES JETS TURBULENTS LIQUIDES

Résumé—Le transfert de masse dans les jets turbulents liquides est discuté à partir du modèle de diffusivité turbulente. On examine la cohérence des données expérimentales disponibles sur les coefficients de transfert de masse. Pour les données qui sont en accord avec la théorie, le paramètre de diffusivité turbulente est déterminé et il est relié aux paramètres hydrodynamiques du système.

STOFFTRANSPORT IN TURBULENTEN FLÜSSIGKEITSSTRAHLEN

Zusammenfassung—Es wurde der Stofftransport in turbulenten Flüssigkeitsstrahlen mit Hilfe des Schein-Diffusionsmodells untersucht. Die verfügbaren experimentellen Daten zur Bestimmung von Stofftransportkoeffizienten solcher Systeme wurden auf ihre Übertragbarkeit hin untersucht. Mit den geeigneten experimentellen Daten wurde der Scheindiffusionsparameter bestimmt und mit den grundlegenden hydrodynamischen Systemparametern in Beziehung gesetzt.

МАССООБМЕН В ТУРБУЛЕНТНЫХ ЖИДКИХ СТРУЯХ

Аннотация—На основе модели вихревой диффузии изучался массообмен в турбулентных струях. Проведен анализ имеющихся экспериментальных данных для коэффициентов массообмена. Из экспериментальных данных, совпадающих с теоретическими результатами, определен параметр вихревой диффузии и установлена его зависимость от основных гидродинамических параметров течения.